D'où $f = r_{(I,\frac{\pi}{2})}$

EX 11 :

1) Montrons que f conserve les distances $\begin{matrix} M_1(x_1,y_1) & M_2(x_2,y_2) \\ M'_1 = f(M_1) & M'_2 = f(M_2) \end{matrix}$

$$M'_1(x'_1,y'_1)tq:\begin{cases} x'_1 = 1-\sqrt{2}+\frac{1}{\sqrt{2}}x_1+\frac{1}{\sqrt{2}}y_1 \\ y'_1 = 1+\sqrt{2}+\frac{1}{\sqrt{2}}x_1-\frac{1}{\sqrt{2}}y_1 \end{cases} \text{ et } M'_2(x'_2,y'_2)tq:\begin{cases} x'_2 = 1-\sqrt{2}+\frac{1}{\sqrt{2}}x_2+\frac{1}{\sqrt{2}}y_2 \\ y'_2 = 1+\sqrt{2}+\frac{1}{\sqrt{2}}x_2-\frac{1}{\sqrt{2}}y_2 \end{cases}$$

$$(M'_1M'_2)^2 = (x'_2-x'_1)^2+(y'_2-y'_1)^2$$

$$= \left[\frac{1}{\sqrt{2}}(x_2+y_2-x_1-y_1)\right]^2+\left[\frac{1}{\sqrt{2}}(x_2-y_2-x_1+y_1)\right]^2$$

$$= \frac{1}{2}\left[(x_2-x_1)^2+(y_2-y_1)^2+2(x_2-x_1).(y_2-y_1)\right]+\frac{1}{2}\left[(x_2-x_1)^2+(y_2-y_1)^2-2(x_2-x_1).(y_2-y_1)\right]$$

$$= (x_2-x_1)^2+(y_2-y_1)^2$$

$$= (M_1M_2)^2$$

d'ou $M'_1M'_2 = M_1M_2$

Par suite f est une isométrie

2) a) soit $M(x,y)$

$$f(M) = M \Leftrightarrow \begin{cases} x = 1-\sqrt{2}+\frac{1}{\sqrt{2}}x+\frac{1}{\sqrt{2}}y \\ y = 1+\frac{1}{\sqrt{2}}x-\frac{1}{\sqrt{2}}y \end{cases}$$

$$\Leftrightarrow \begin{cases} \sqrt{2}x = \sqrt{2}-2+x+y \\ \sqrt{2}y = \sqrt{2}+x-y \end{cases}$$

$$\Leftrightarrow \begin{cases} (\sqrt{2}-1)x-y = \sqrt{2}-2 \\ x-(1+\sqrt{2})y = -\sqrt{2} \end{cases}$$

$$\Leftrightarrow \begin{cases} x-(1+\sqrt{2})y+\sqrt{2} = 0 \\ x-(1+\sqrt{2})y+\sqrt{2} = 0 \end{cases}$$

$$\Leftrightarrow x-(1+\sqrt{2})y+\sqrt{2} = 0$$

D'où l'ensemble des points invariant par f est la droite $\Delta : x-(1+\sqrt{2})y+\sqrt{2} = 0$

Qarray.tn - Aqra Blech - Qarray.tn